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UIMO
Unified International Mathematics Olympiad

UNIFIED INTERNATIONAL MATHEMATICS OLYMPIAD (UPDATED)

CLASS - 9
Question Paper Code : UM9279

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A,B,C,D	B,C	A,D	A,B,D	Delete	C	D	D	Delete	A
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C	B	D	B	C	B	C	D	D	B

EXPLANATIONS

MATHEMATICS - 1

1: (B) If $f(x) = (x^{100} + 3x^{99} + k)$ is divided by $(x + 1)$ then remainder is (-1)

$\therefore$ $f(-1) = [(-1)^{100} + 3(-1)^{99} + k] = 0$

$1 - 3 + k = 0$

$k = 2$

2: (C) Given $5x + 12x + 13x = 60$ cm

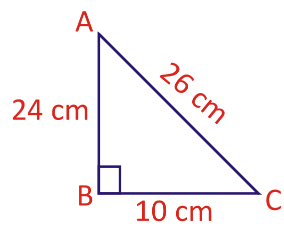
$30x = 60$ cm

$x = \dfrac{60\,\text{cm}}{30} = 2$ cm

$\therefore$ $5x = 5 \times 2$ cm $= 10$ cm, $12x = 12 \times 2$cm $= 24$ cm

and $13x = 13 \times 2$ cm $= 26$ cm

$24^2 + 10^2 = 576 + 100 = 676 = 26^2$



$\therefore$ 10 cm, 24 cm & 26 cm are the sides of a right angled triangle

Longest altitude is the perpendicular to the smallest side

$\therefore$ Longest altitude = 24 cm

3. (C) $\sqrt{21-4\sqrt{5}+8\sqrt{3}-4\sqrt{15}}$

$$=\sqrt{21-2\sqrt{20}+2\sqrt{48}-2\sqrt{60}}$$

$$=\sqrt{12+4+5-2\sqrt{4}\times\sqrt{5}+2\sqrt{4}\times\sqrt{12}-2\sqrt{12}\times\sqrt{5}}$$

$$\sqrt{\left(\sqrt{12}\right)^2+\left(\sqrt{4}\right)^2+\left(-\sqrt{5}\right)^2+2\sqrt{4}\left(-\sqrt{5}\right)+2\sqrt{4}\times\sqrt{12}+2\sqrt{12}\times\left(\sqrt{5}\right)}$$

$$\sqrt{\left(\sqrt{12}+\sqrt{4}-\sqrt{5}\right)^2}=\left(2\sqrt{3}+2-\sqrt{5}\right)$$

4: (B) ABCD is a square and diagonal bisect each other at 'O' (0, 0)

5: (A) Volume of cylinder = $\pi r^2 h$

$$=\frac{22}{\cancel{7}_1}\times\cancel{7}_1\times 7\times 15\,\text{cm}^3$$

$$= 2310\,\text{cm}^3$$

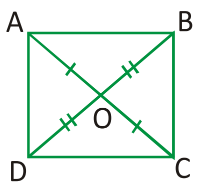
06. (A) Volume of cylinder = $\pi r^2 h$

$$\Rightarrow 66=\frac{22}{7}\times\frac{1}{20}\times\frac{1}{20}\times h$$

$$\Rightarrow \frac{66\times 7\times 20\times 20}{22}=h$$

$$\Rightarrow h = 8400\text{ cm or } 84\text{ m}$$

7: (B) ABCD is a parallelogram



[$\therefore$ diagonals are bisecting each other]

$\therefore$ BC || AD & AB = CD

8: (C) Given $a^3 : b^3 = 27 : 8$

$$\Rightarrow \frac{a^3}{b^3}=\frac{27}{8}=\left(\frac{3}{2}\right)^3$$

$$\Rightarrow \left(\frac{a}{b}\right)^{\cancel{3}}=\left(\frac{3}{2}\right)^{\cancel{3}}$$

$\therefore$ a : b = 3 : 2

9: (B) Area of parallelogram ABCD = 2 [Area of ΔABL + area of ΔDCL]

= 2(15 cm^2 + 28 cm^2)

= 2 × 43 cm^2

= 86 cm^2

10: (D) PQRS is the cyclic quadrilateral

$\therefore$ $\angle$RSP = 180° – $\angle$PQR = 40°

In ΔOSR, OS = OR $\Rightarrow$ $\angle$ORS = $\angle$RSO = 40°

$\therefore$ x = $\angle$ORS + $\angle$RSO = 80°

11: (C) In a triangle third side must be less than sum of the other two sides and greater than difference of two sides.

$\therefore$ Third side must be greater than 3 cm and less than 11 cm

12: (C) LHS = $\frac{1}{\left(4-\sqrt{15}\right)}\times\frac{\left(4+\sqrt{15}\right)}{\left(4+\sqrt{15}\right)}$

$$-\frac{1}{\left(\sqrt{15}+\sqrt{14}\right)}\times\frac{\left(\sqrt{15}-\sqrt{14}\right)}{\left(\sqrt{15}-\sqrt{14}\right)}$$

$$-\frac{1}{\left(\sqrt{14}+\sqrt{13}\right)}\times\frac{\left(\sqrt{14}-\sqrt{13}\right)}{\left(\sqrt{14}-\sqrt{13}\right)}-\frac{1}{\left(\sqrt{13}+\sqrt{12}\right)}$$

$$\times\frac{\left(\sqrt{13}-\sqrt{12}\right)}{\left(\sqrt{13}-\sqrt{12}\right)}-\frac{1}{\left(\sqrt{12}+\sqrt{11}\right)}\times\frac{\left(\sqrt{12}-\sqrt{11}\right)}{\left(\sqrt{12}-\sqrt{11}\right)}$$

$$-\frac{1}{\left(\sqrt{11}+\sqrt{10}\right)}\times\frac{\left(\sqrt{11}-\sqrt{10}\right)}{\left(\sqrt{11}-\sqrt{10}\right)}$$

$$-\frac{1}{\left(\sqrt{10}+3\right)}\times\frac{\left(\sqrt{10}-3\right)}{\left(\sqrt{10}-3\right)}$$

$= \left(4+\sqrt{15}\right) - \left(\sqrt{15}-\sqrt{14}\right)$

$- \left(\sqrt{14}-\sqrt{13}\right) - \left(\sqrt{13}-\sqrt{12}\right)$

$- \left(\sqrt{12}-\sqrt{11}\right) - \left(\sqrt{11}-\sqrt{10}\right) - \left(\sqrt{10}-3\right)$

$= 4 + \cancel{\sqrt{15}} - \cancel{\sqrt{15}} + \cancel{\sqrt{14}} - \cancel{\sqrt{14}} + \cancel{\sqrt{13}}$

$- \cancel{\sqrt{13}} + \cancel{\sqrt{12}}$

$- \cancel{\sqrt{12}} + \cancel{\sqrt{11}} - \cancel{\sqrt{11}} + \cancel{\sqrt{10}} - \cancel{\sqrt{10}} + 3$

$= 7$

13: (B) Let OM = x

In ΔBOM and ΔABM, $BM^2 = OB^2 - OM^2 = AB^2 - AM^2$

$\therefore\ 5^2 - x^2 = 6^2 - (5-x)^2$

$25 - x^2 = 36 - (25 - 10x + x^2)$

$25 - x^2 = 36 - 25 + 10x - x^2$

$25 - 11 - x^2 + x^2 = 10x$

$x = 1.4$ cm

14: (D) Given ABCD is a parallelogram

$\therefore$ AB = CD

$\Rightarrow \dfrac{AB}{2} = \dfrac{CD}{2}$

$\Rightarrow$ AX = CY and AX || CY

$\therefore$ AXCY is a parallelogram

15: (D) $s = \dfrac{a+b+c}{2} = \dfrac{6cm+9cm+5cm}{2} = 10$ cm

Area of triangle

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{10\times4\times1\times5\,cm^4}$

$= \sqrt{200\,cm^4}$

$= 10\sqrt{2}\ cm^2$

16: (C) $\angle A + \angle C + \angle E + \angle G = 360°$ & $\angle B + \angle D + \angle F + \angle H = 180°$

$\therefore\ \angle A + \angle B + \angle C + \angle D + \angle E + \angle F + \angle G + \angle H = 360° + 360° = 720°$

17: (A) AOB is an isosceles right angled triangle

18: (A) On x-axis all y-coordinates are zero

$\therefore$ $2x + 3(0) = 6$

$x = \dfrac{6}{2} = 3$

$\therefore$ The line $2x + 3y = 6$ cuts x-axis at (3, 0)

19: (A) Given $\pi r^2 = 616\ cm^2$

$\dfrac{22}{7} \times r^2 = 616\ cm^2$

$r^2 = \cancel{616}^{\cancel{56}\,28}\ cm^2 \times \dfrac{7}{\cancel{22}_{\cancel{2}\,1}} = 2\times2\times7\times7\,cm^2$

$r^2 = (14cm)^2$

$r = 14$ cm

$l = \sqrt{h^2+r^2} = \sqrt{48^2+14^2}$

$= \sqrt{2304+196}$

$= \sqrt{2500}$

$l = 50$ cm

TSA of cone = $\pi r(l + r)$

$= \dfrac{22}{\cancel{7}_1} \times \cancel{14}^2\ cm\ (50 + 14)\ cm$

$= 44\ cm \times 64\ cm = 2816\ cm^2$

20: (C) $p^3(q-r)^3 + q^3(r-p)^3 + r^3(p-q)^3$

$= [p(q-r)]^3 + [q(r-p)]^3 + [r(p-q)]^3$

$= (pq - pr)^3 + (qr - pq)^3 + (pr - qr)^3$

Let $a = pq - pr$, $b = qr - pq$ and $c = pr - qr$

$\therefore$ $a + b + c = pq - pr + qr - pq + pr - qr = 0$

$\therefore$ $a^3 + b^3 + c^3 = 3abc$

$\therefore$ $p^3(q-r)^3 + q^3(r-p)^3 + r^3(p-q)^3 = 3pqr$

$(p - r)(q - r)(r - p)$

21: (B) $y - 10 = 0 \Rightarrow y = 10$ line is paralled to X-axis

22: (D) Given $\frac{1}{\sqrt{x}}+\frac{1}{x+\sqrt{x}}=1$

$$\frac{x+\sqrt{x}+\sqrt{x}}{\sqrt{x}\left(x+\sqrt{x}\right)}=1$$

$$x+2\sqrt{x}=\sqrt{x}\left(x+\sqrt{x}\right)$$

$$\cancel{x}+2\sqrt{x}=x\sqrt{x}+\cancel{x}$$

$$x\sqrt{x}=2\sqrt{x}$$

$$x=\frac{2\cancel{\sqrt{x}}}{\cancel{\sqrt{x}}}=2$$

23: (C) In ΔOPQ, OP = OQ $\Rightarrow$ $\angle$OQP = $\angle$OPQ = m

& $\angle$POQ = 2$\angle$R = $2l$

In ΔOPQ, m + m + $2l$ = 180°

$2(l + \text{m}) = 180°$

$$l+\text{m}=\frac{180°}{2}=90°$$

24: (B) Given r = 4 cm & h = 15 cm

Given volume of x spheres = volume of cylinder

$$\Rightarrow x\times\frac{4}{3}\cancel{\pi}\text{r}^3=\cancel{\pi}\text{R}^2\text{h}$$

$$x\times\frac{4}{3}\times\left(\frac{1}{2}\text{cm}\right)^3=(4\text{ cm})^2\times 15\text{ cm}$$

$$x=16\text{cm}^2\times 15\text{cm}\times \cancel{8}^2\times\frac{3}{\cancel{4}_1\text{ cm}^3}$$

= 1440

25: (C) LHS

$$=\sqrt{(3\text{a}+2\text{b}-\text{c}+\text{d})^2-12\text{a}(2\text{b}-\text{c}+\text{d})}$$

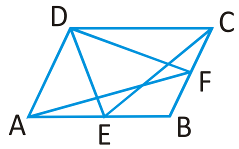
$$=\sqrt{\left[(3\text{a})+(2\text{b}-\text{c}+\text{d})\right]^2-4\times(3\text{a})(2\text{b}-\text{c}+\text{d})}$$

$$=\sqrt{\left[3\text{a}-(2\text{b}-\text{c}+\text{d})\right]^2}$$

[$\because$ (a+b)^2– 4ab = (a–b)^2]

= (3a – 2b + c – d)

26: (C) Given Area of ΔADF = 24 cm^2



$\therefore$ Area of the parallelogram ABCD = 2 × Area of ΔADF = 48 cm^2

[$\therefore$ A triangle and a parallelogram lie between same parallel lines and having common base then area of parallelogram is twice the triangle area]

$\therefore$ Area of ΔCDE = $\frac{1}{2}$ × 48 cm^2 = 24 cm^2

27: (A) Given $x + y = 12$ & $xy = 27$

$\therefore$ $(x - y)^2 = (x + y)^2 - 4xy$

$= 12^2 - 27 \times 4$

$= 144 - 108$

$(x - y)^2 = 36$

$x - y = \sqrt{36} = 6$

cubing on both sides

$x^3 - y^3 - 3xy(x - y) = 216$

$x^3 - y^3 - 3 \times 27 \times 6 = 216$

$x^3 - y^3 = 216 + 486$

$x^3 - y^3 = 702$

28: (A)

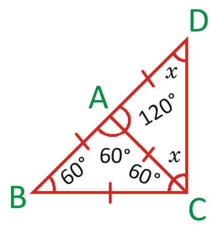
$$\begin{array}{l|l|l}
2x+3 & 4x^3+20x^2+33x+18 & 2x^2+7x+6 \\
 & \underline{\overset{(-)}{4x^3}+\overset{(-)}{6x^2}} & \\
 & \quad 14x^2+33x+18 & \\
 & \quad \underline{\overset{(-)}{14x^2}+\overset{(-)}{21x}} & \\
 & \qquad 12x+18 & \\
 & \qquad \underline{12x+18} & \\
 & \qquad\quad 0 &
\end{array}$$

$2x^2 + 7x + 6 = 2x^2 + 4x + 3x + 6$

$= 2x(x + 2) + 3(x + 2)$

$= (x + 2)(2x + 3)$

29: (C) In a ΔACD, $\angle$CAD = 180° – 60° = 120°



In ΔACD, $AD = AC \Rightarrow \angle ACD = \angle D = x$

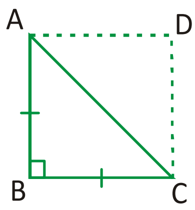
$x + x + 120° = 180°$

$2x = 60°$

$x = 30°$

$\therefore$ $\angle BCD = 60° + x = 60° + 30° = 90°$

30: (B) Area of square ABCD $= \frac{1}{2}\ d^2$



$= \frac{1}{\cancel{2}_1} \times \cancel{12}^6 \times 12\ cm^2$

$= 72\ cm^2$

$\therefore$ Area of $\Delta ABC = \frac{1}{2} \times 72\ cm^2 = 36\ cm^2$

MATHEMATICS - 2

31: (A, B, C, D)

$x^6 - 1 = (x^3)^2 - 1^2$

$= (x^3 - 1)(x^3 + 1)$

$= (x - 1)\ (x^2 + x + 1)(x + 1)\ (x^2 - x + 1)$

But $x^6 - 1 = (x^2)^3 - 1^3$

$= (x^2 - 1)\ (x^4 + x^2 + 1)$

32: (B, C)

Options B & C are true

33: (A, D)

Given $(x - 1)$ is a factor of $P(x) = x^3 + 10x^2 + px + q$

$\therefore$ $P(1) = 0 \Rightarrow 1 + 10(1) + p(1) + q = 0$

$\therefore$ $p + q = -11 \rightarrow (1)$

Given $P(-2) = 0 \Rightarrow (-2)^3 + 10(-2)^2 + P(-2) + q = 0$

$-8 + 40 - 2p + q = 0$

$2p - q = 32 \rightarrow (2)$

eq (2) + (1) $\Rightarrow (2p - q) + (p + q) = 32 + (-11)$

$2p - \cancel{q} + p + \cancel{q} = 21$

$3p = 21$

$p = \frac{21}{3} = 7$

$7 + q = -11$

$\Rightarrow q = -11 - 7 = -18$

34: (A, B, D)

Third angle of triangle = 180° – 117° = 63°

For option A : 64° + 53° = 117°

For option B : 63° + 54° = 117°

For option C : 63° + 63° + 63° ≠ 180°

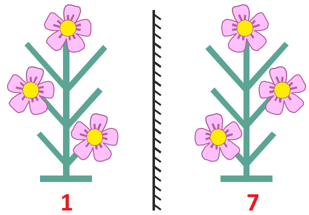
For option D : 69° + 48° = 117°

35. (Delete)

REASONING

36. (C) Second word is the mirror image of the first word.

37. (D) 1 and 7



38. (D) (A) $29 - 18 \div 6 < 36 \div 6 \times 4$

$29 - 2 < 6 \times 4$

$27 < 24$ (Wrong)

(B) $18 + 12 \div 4 > 7 + 8 \times 2$

$18 + > 7 + 16$

$21 > 23$ (Wrong)

(C) $32 + 6 \div 2 < 6 - 7 \times 2$

$32 + 3 < 6 - 15$

$35 < -8$ (Wrong)

(D) $31 + 1 - 2 < 4 + 6 \times 7$

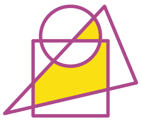
$30 < 4 + 42$

$30 < 82$ (Correct)

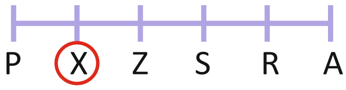
39. (Delete)

40. (A) All except scorpion remaining animals live in water.

41. (C) One dot occupies the region which is common to the circle and triangle and the other dot occupies the region which is common to the triangle and square. Out of all the answer figures, only answer figure (C) possesses a region which is common to the circle and triangle and a region which is common to the triangle and square.

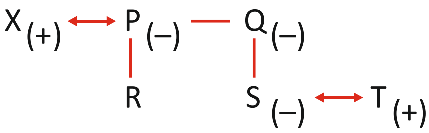


42. (B)



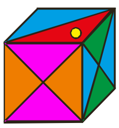
43. (D)

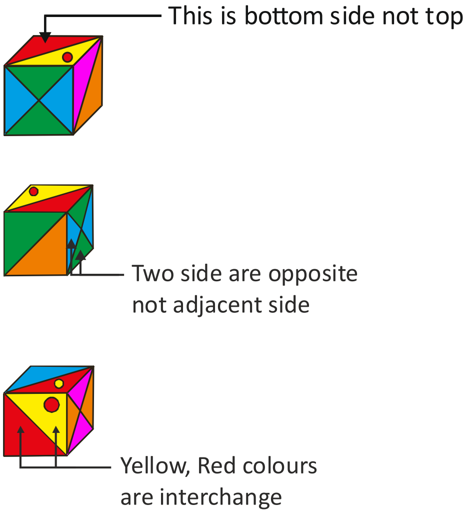
44. (B) Mother-in-law



'Q' is Mother-in-law to 'T'.

45. (C)





CRITICAL THINKING

46. (B) Weight B requires a force equal to 5 Kg whereas A requires a force equal to 10 Kg.

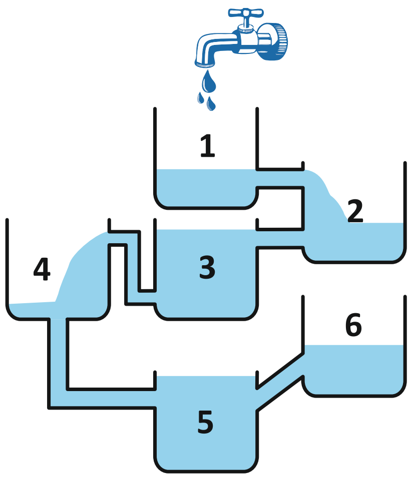
Single pulley questions are relatively straight forward. If the pulley is fixed, then the force required is equal to the weight. If the pulley moves with the weight then the force is equal to half of the weight. Another way of thinking about this is to divided the weight by the number of sections of rope supporting it to obtain the force needed to lift it. In A there is only one section of rope supporting the weight, so 10/1 = 10 Kg required to lift the weight. In B there are two sections of rope supporting the weight, so 10/2 = Kg required to lift it.

47. (C) 5

48. (D) Either statement can't give the answer.

From both statements we can get the relationship as Pooja is Neeraj's sister but it cannot be clearly said that Shubham & Meenal are Pooja's children or not & Shivani is Neeraj's daughter, because there can be other siblings of Neeraj and Pooja also.

49. (D) Tank 5 will fill up first



50. (B) Clearly, damage to crops due to high temperature may have resulted in a short supply of vegetables and hence an increase in their prices